EXPERIMENTAL STUDY OF FLOW THROUGH TRAPEZOIDAL WEIR CONTROLLED UNDER A SEMI-CIRCULAR GATE

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ABSTRACT
Different parameters of a weir model have a great effect on the discharge coefficient. In this experimental study the effect of varying angle of a trapezoidal weir coupled with a below semi-circular gate is determined. The result showed that the higher the value of \( \theta \) the higher the coefficient of discharge. The respective average discharge coefficient \( C_{dave} \) of the block model and the trapezoidal weir models are: 0.48031, 0.48880, 0.49565, 0.49647, 0.49892 and 0.49934. As such the trapezoidal weir with \( \theta = 60^\circ \) has the highest value of average discharge coefficient \( C_{dave} = 0.49934 \). Hence the most efficient. Linear and nonlinear regression analysis were used to generate mathematical equations that can be used to predict the flow rate \( Q \) for the combined weir-gate structure and the discharge coefficient \( C_d \) of the most efficient model with \( \theta = 60^\circ \) respectively. The discharge coefficient for the most efficient weir model was found to be 3.81% more than that of the block model (with rectangular weir). The predicted coefficient of discharge \( C_{d(pre)} \) for the most efficient model was also found to be in good agreement with the observed discharge coefficient \( C_{d(obs)} \) with a percentage error in the range of \( \pm 0.4\% \).

Keywords: coefficient of discharge, weir angle, trapezoidal weir

INTRODUCTION
Weir can be defined as an overflow structure placed across a channel to measure or control the level of water at the upstream side of the channel by discharging the excess flow and it can also be used for raising and dissipation of energy which can helps in reducing or preventing damages to hydraulic structure e.g dams (Zakwan and Khan, 2020).

There are different types of weir depending upon their shapes, nature of discharge and width of crest. According to Rajput (2008) the most important types of weir are rectangular and cippoletti weirs according to the weirs’ geometry, ordinary and submerge weirs based on the nature of discharge and broad-crested and narrow crested weirs as a result of the crest width. Among these weirs, sharp-crested and broad-crested are the two commonly used types of weirs (Piratheepan et al., 2006). One of the disadvantage of weirs is that they required periodically cleaning of sediments and waste (Ferro, 2000).

Another type of weir is a circular thin plate weir which is usually positioned in a vertical thin plate and put in at right angle (90\(^\circ\)) to the sides and bottom of the channel. Circular weirs have advantage that the crest can be turned and not need to be leveled. Sharp crested circular weirs are fully contracted so that the bed and sides of the channel could sufficiently provide a relative remote distance to have no influence on the development of the nappe (Radd et al,2014).

Moreover, weirs have been generally applied in measurement of flow, diversion of flow and in open channels for flow control (Kumar et al.,2011). As such, Sediment usually accumulates at the bottom of weirs, so to maintain accurate flow measurement the weir pool should be clean. A sluice gate is a bottom opening in a wall, commonly used in control of rivers and channel flows. One disadvantage of the sluice gates is they retained the floating materials. In order to minimize this problem, weirs and gates are combined together in one device yielding a simultaneous flow over the weir and below the gate. The combined weir and gate systems can be used in minimizing sedimentsations and depositions (Khassaf and Habeeb, 2014). This inform our decision in utilizing the combined effects of trapezoidal weir and semi-circular gate in the present study.

Hayawi et al (2008) investigated the effect of combined flow over triangular weir by varying the notch angles and below...
rectangular gate, however they found that the discharge coefficient increases with increase in notch angle \( \theta \). Saad and Fattouh (2017) investigate the influence of varying weir openings on hydraulic characteristics of flow over weir with circular openings. They varies the diameters of the opening (D) and heights from the bed (Z). They concluded that the coefficient of discharge increases with decrease in \((\frac{D}{Z})\) for weirs with the same number of openings and also the smaller the number of opening the higher the discharge. Ismail (2012) studied the effect of hydraulic and geometrical parameters on the coefficient of discharge on sharp crested trapezoidal weir with below rectangular sluice gate and proposed two equations to evaluate it. Naori and Hemmati (2020) study discharge coefficient in the combined weir-gate structure. They investigated the discharge coefficient of rectangular compound broad crested weir. They concluded that increasing the gate opening and gate width decrease the discharge coefficient as well as increasing the central height of the weir.

Rafi et al. (2018) studied the effect of parabolic weir over flow and gate under flow. They compared the result obtain from their study and that of a regular shape of combine weir and gate. They found that the weir and gate cross sectional area of flow have significant effect on coefficient of discharge of hydraulic structure. The discharge coefficient increases with increase in weir cross sectional area and but decreases with increase in cross sectional area of the gate. Thus, the main objective of this study is to investigate the effect of varying angle of a trapezoidal weir coupled with a below semi-circular gate in a flow channel.

**THEORETICAL ANALYSIS**

Figure 1 shows the free flow over a weir (skimming) and through the semi-circular gate.

The total flow through the gate and above the weir is given by:

\[ Q_T = Q_w + Q_g \]

Where \( Q_T \) = total flow through the gate and above the weir, \( Q_w \) = flow over the weir and \( Q_g \) = flow through the gate.

\[ Q_w = \frac{2}{3} C_{dr} L \sqrt{2gh^2 + \frac{8}{15} C_{dt} \tan \frac{\theta}{2} \sqrt{2gh^2}} \]

\[ Q_g = \frac{1}{8} \pi D^2 \left( 2gH \right)^{\frac{5}{2}} \]

The theoretical discharge equations for designing the combined trapezoidal weir and below semi-circular gate models may be derived by using linear combination method as expressed by Eqn. (4)

\[ Q_{theo} = \left\{ \left( \frac{2}{3} C_{dr} L \sqrt{2gh^2 + \frac{8}{15} C_{dt} \tan \frac{\theta}{2} \sqrt{2gh^2}} \right) + \frac{1}{8} \pi D^2 \left( 2gH \right)^{\frac{5}{2}} \right\} \]

Where, \( Q_{theo} \) = total flow over weir and through gate, \( g \) = gravitational acceleration, \( C_{dr} \) = discharge coefficient for simple rectangular weir, \( C_{dt} \) = discharge coefficient for simple triangular weir, \( D \) = diameter of semi-circular gate, \( L \) = weir crest length, \( H \) = total head (\( d + y + h \)), \( h \) = water head above weir of crest, \( y \) = the distance between upper edge of the gate and lower edge of the weir, \( d \) = height of gate and \( \frac{\theta}{2} \) = weir angle.

\( C_{dt} \) Value can be estimated from the following empirical relationship (Jan et al., 2006):

\[ C_{dt} = 0.6085 - 0.05250 + 0.02135\theta^2 \]

Where \( \theta \) is in radians.

Also, according to French, 1986 coefficient of discharge of a simple rectangular sharp crested weir can be written as

\[ C_{dt} = 0.6085 - 0.05250 + 0.02135\theta^2 \]
Experimental Study...

\[ C_{dr} = \frac{0.611 + 2.236 \frac{\rho}{1 + 3.3 (\frac{\rho}{1 + 3.3})^0.7}}{1 + 3.3 (\frac{\rho}{1 + 3.3})^0.7} + \frac{0.075 - 0.011 (\frac{\rho}{1 + 3.3})^{1.46}}{1 + 3.3 (\frac{\rho}{1 + 3.3})^{1.46}} \]  
\[ b \]
\[ \sigma \]
\[ q \]  
\[ \gamma \]
\[ d \]
\[ \theta \]
\[ \rho \]
\[ \mu \]  
\[ Q_{act} = f_{1}(H, b, h, D, h, g, \rho, \mu, \sigma) \]
\[ Q_{act} = f_{2}(H, b, h, D, h, g, \frac{\mu}{\rho \omega^2}, \frac{\sigma}{\rho \omega^2}) \]
\[ C_d = f_{3}(\frac{H}{D}, \frac{b}{D}, \frac{g}{g^2}, \frac{H}{H}, \frac{\mu}{\rho \omega^2}, \frac{\sigma}{\rho \omega^2}) \]

Where \( B \) = channel width, \( P \) = weir height and \( b \) = width of weir opening.

However, it should be noted here that for rectangular weir (block model), it is assume the shape of a rectangle because there is no introduction of an angle to the side of the weir which is perpendicular to the crest as such the angle is taken as zero degree. Therefore the triangular component, \( \frac{8}{15} C_{d} \tan \frac{\theta}{2} \sqrt{2 \rho h^3} \), is eliminated. Hence Eqn. (3.11) becomes:

\[ Q_{theo} = \left\{ \begin{array}{ll}
\frac{2}{3} C_{d} b \sqrt{2 \rho h^3} + \frac{1}{8} \pi D^2 \left(2gH\right)^{\frac{5}{2}} & \\
\end{array} \right. \]  
\[ (7) \]

Where \( b \) = width of weir opening.

The selected parameters that have influence on the weir coefficient of discharge can be functionally expressed as follow according to Khassaf and Habeeb (2014)

\[ Q_{act} \sqrt{\frac{g}{d^2}s} = f_{1}(\frac{H}{D}, \frac{b}{D}, \frac{h}{D}, \frac{D}{D}, \tan \frac{\theta}{2}, \text{Re}, \text{We}) \]  
\[ (8) \]

Where \( H \) = total head, \( h \) = head of water over weir, \( D \) = the gate diameter, \( y \) = vertical distance between the lower edge of the weir and the upper edge of the gate opening, \( L \) = weir crest length, \( B \) = channel width, \( \theta \) = trapezoidal angle, \( \text{Re} \) = Reynolds number, \( \text{We} \) = Weber number. \( \text{Re} \) and \( \text{We} \) can be represented by one dimensional variable \( h \) or \( H \) according to Ackers, 1978. Using Buckingham \( \pi \)-theorem, the functional relationship in eqn. (8) can be written in terms of \( C_d \) as follows:

\[ C_d = f_{2}(\frac{H}{D}, \frac{b}{D}, \tan \frac{\theta}{2}, \text{Re}, \text{We}) \]  
\[ (9) \]

For flow through the rectangular weir and below semicircular gate, the discharge \( Q_{act} \) can be expressed by the functional relationship

\[ Q_{act} = f_{1}(y, b, D, h, \rho, \mu, \sigma) \]  
\[ (10) \]

Where, \( \rho \) = density of water, \( \mu \) = dynamic viscosity and \( \sigma \) = surface tension. Based on Eqn. (10) and using dimensional analysis (Buckingham-\( \pi \) Theorem) the functional relationship becomes:

\[ Q_{act} \sqrt{\frac{g}{d^2}s} = f_{2}(\frac{H}{D}, \frac{b}{D}, \frac{h}{D}, \frac{D}{D}, \frac{\mu}{\rho g^2}, \frac{\sigma}{\rho g^2}) \]  
\[ (11) \]

But \( \frac{\mu}{\rho g^2} = \frac{1}{\text{Re}} \) where \( \text{Re} \) = Reynolds number

And \( \frac{\sigma}{\rho g^2} = \frac{1}{\text{We}} \) where \( \text{We} \) = Weber number

Hence in terms of \( C_d \), Eqn. (11) can be written as

\[ C_d = f_{3}(\frac{H}{D}, \frac{b}{D}, \frac{\mu}{\rho g^2}, \frac{\sigma}{\rho g^2}) \]  
\[ (12) \]

The effect of Reynolds number and Weber number is assumed to be negligible for the combined weir-gate structure except at a very low head (Khassaf et al 2013).

3.0 Experimental Set up

3.1 Model Description

Six models were fabricated from plywood. All of the six models have same dimensions of weir thickness \( t_w = 4 \text{mm} \); length of weir crest \( L = 20 \text{cm} \); height of weir \( P = 24 \text{cm} \); crest height \( h_c = 4 \text{cm} \); Distance between the lower edge of the weir and the upper edge of the semi-circular gate \( y = 10 \text{cm} \); diameter of gate \( D = 20 \text{cm} \) and height of the semi-circular gate \( d = 10 \text{cm} \). The weir angles \( \theta = 7.5^\circ, 15^\circ, 30^\circ, 45^\circ \) and \( 60^\circ \) varied for each of the five trapezoidal weirs and then a rectangular weir (block model) making a total of six weirs models. The combined weir-gate models that were fabricated and tested are shown in figure 3. Table 1 shows the respective dimensions of the models.
Table 1: Dimensions of weir models that were fabricated and tested.

<table>
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<th>P(cm)</th>
<th>h(cm)</th>
<th>D(cm)</th>
<th>d(cm)</th>
<th>y(cm)</th>
<th>L(cm)</th>
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The discharge coefficient $C_d$ will be obtained as a ratio of the actual discharge to the theoretical discharge. The actual discharge is that which was measured by the weir-gate structure.

METHODS
The experiments were carried out at the Hydraulic laboratory of Department of Water Resources and Environmental Engineering, Ahmadu Bello University (ABU), Zaria. The experimental set consists of a horizontal flume of dimension 6 m long by 0.3 m width and 0.3 m depth. The rectangular flume was levelled horizontally (Slope, $S_o=0$) and then the weir model set at right angle to the direction of flow was installed at 3 m from the inlet channel for uniform flow as suggested by Munta, (2010) and glued within the walls of the flume in order to prevent leakage of water at the walls of the flume. Water from the flume storage basin was then pumped to the flume channel by a centrifugal pump through a pipe of 150 mm which was regulated by a valve. The water was allowed to continue flowing through the gate and above the crest of the weir until the flow became steady then readings were taken. The flume...
measured discharge in terms of mass flow rate, it was then converted to volumetric discharge by dividing with the density of water. A movable point gauge with ±1 mm sensitivity was used to measure the water head over the crest of the weir. The same situation is observed in all the experimental runs. Figure 2 shows the flume that was used for conducting the experiment.

Plate 2: The flume used for conducting the experiment

4.0 Results and Discussion:

4.1 Variation of $C_d$ with $\frac{H}{D}$

The effect of the ratio of total head to the gate diameter $\frac{H}{D}$ on the coefficient of discharge $C_d$ was studied from the tests conducted on the combined weir-gate structure. Figure 4 to 9 shows the variation of $C_d$ with $\frac{H}{D}$ for the respective trapezoidal weir angle $\frac{\theta}{2} = 7.5^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ$ and the block model (rectangular weir).

Figure 4: Variation of $C_d$ with $\frac{H}{D}$ for trapezoidal weir angle $\frac{\theta}{2} = 7.5^\circ$

Figure 5: Variation of $C_d$ with $\frac{H}{D}$ for trapezoidal weir angle $\frac{\theta}{2} = 15^\circ$
From the graphs of figures 4 to 9 it can be observed that the value of $C_d$ increases with a corresponding increase in the value of $H/D$. However it can also be clearly observed from the respective graphs, with the exception of figure 6 and 8, that there is a sharp decrease in the value of $C_d$ corresponding with the value of $H/D$ for the fourth run despite the obvious increase in the value of $H/D$ and then a sudden increase in the $C_d$ value with respect to $H/D$ for the final (fifth) run. This sharp decrease and sudden increase in the value of $C_d$ with respect to the value of $H/D$ is an indication that the coefficient of discharge, $C_d$, is not always entirely a function of the ratio of the total head to the gate diameter, $H/D$, that is $C_d$ cannot be determined solely by the value of $H/D$. Hence a nonlinear relationship. This shows that there is a good agreement between this research with that of Khassam and Habeeb (2014) which says that the coefficient of discharge increases exponentially with in level of water above the crest. Also from the figures above, it can be seen that the narrower the weir angle, the lower the discharge coefficient and the wider the weir angle, the higher the discharge coefficient. It can also be
observed that the value of coefficient discharge for closer angles does not vary much because of close proximity of flow area of the trapezoidal weirs as view for the cases of $\theta_2=7.5^\circ$ and $15^\circ$.

Figure 10 shows the variation of $C_d$ with $\frac{H}{D}$ for all the weir models.

**Variation of $C_d$ with $Q_{theo}$**

The variation of $C_d$ with $Q_{theo}$ for all the weir models is shown in figure 17. It can be observed from the figure that the trend of variation of the values of $C_d$ with respect to $Q_{theo}$ is the same as that with respect to $\frac{H}{D}$. This is so because for a constant value of weir angle $\theta_2$ as well as the block model weir opening, $Q_{theo}$ is a function of the total head, $H$; and subsequently $\frac{H}{D}$, that is $Q_{theo}$ increases with increase in the value of $H$.

Figure 17: Variation of $C_d$ with $Q_{theo}$ for all the weir models.
Variation of $C_{dave}$ with $\frac{\theta}{2}$

The average coefficient of discharge $C_{dave}$ ($\frac{\Sigma C_d}{n}$, where $n$ is the number of runs) as observed from the results obtained from the conducted experiment, increases with increase in weir angle, that is the wider the angle the higher the average discharge coefficient and the narrower the angle the lower the average discharge coefficient. Figure 18 shows the variation of $C_{dave}$ with $\frac{\theta}{2}$.

![Figure 18: Variation of $C_{dave}$ with $\frac{\theta}{2}$](image)

From figure 18 it can be observed that $C_{dave}$ increases with increase in $\frac{\theta}{2}$. For the block model, which is a rectangular weir, there is no introduction of an angle to the side of the weir which is perpendicular to the crest. Hence the angle is taken as zero degree.

Mathematical modelling of flow rate “Q” and discharge coefficient “$C_d$”:

The following expressions were developed using the results obtained from the experiment

$$Q = Kh^n$$  \hspace{1cm} (14)

Equation (14) is the discharge – head relationship which can be used to calculate the flow rate through the combined sharp-crested weir-gate models.

$$C_d = \alpha \left( \frac{H}{D} \right)^{\beta} \left( \frac{h}{D} \right)^{\gamma} \tan \left( \frac{\theta}{2} \right)^{\epsilon}$$  \hspace{1cm} (15)

Equation (15) shows the non-linear variation of $C_d$ with $\frac{H}{D}$, $\frac{h}{D}$ and $\tan \left( \frac{\theta}{2} \right)$ which can be applied in determining the coefficient of discharge of any of the combined sharp-crested trapezoidal weir and semi-circular gate models. The parameters of the above equations that is $k$ and $n$ in eqn.14, $\alpha$, $\beta$, $\gamma$ and $\theta$ in eqn.15 were determined using regression analysis.

4.4.1 Empirical Relation of Flow Rate, $Q$

The following expression was developed using the experimental results

$$Q = 0.662H^{1.260}$$  \hspace{1cm} (16)

Equation (16) is the discharge-head relationship which can be applied to calculate the discharge over the broad-crested weir.

Empirical Relation of Coefficient of Discharge, $C_d$

Based on eqn. 4.02, multiple non-linear regression analysis was used to correlate $C_d$ with $\frac{H}{D}$, $\frac{h}{D}$ and $\tan \left( \frac{\theta}{2} \right)$ in an empirical power relation

$$C_d = 0.526 \left( \frac{H}{D} \right)^{0.093} \left( \frac{h}{D} \right)^{0.034} \tan \left( \frac{\theta}{2} \right)^{0.040}$$  \hspace{1cm} (17)

Eqn. (17) can be used to predict the coefficient of discharge for the most efficient model. With correlation coefficient ($R^2$) value obtained as 0.992. Figure 24 shows the relationship between observed $C_d$ and predicted $C_d$ for the most efficient model as obtained from the experimental results.
Error distribution:
The error distribution in predicting the experimentally observed $C_d$ is shown in figure 26. It can be observed from the graph that the percentage error is within ± 0.4%. Therefore, the most efficient combined hydraulic measuring device presented in this study is said to be an accurate measuring device. The percentage error can be computed using the following expression:

$$\% \text{ Error} = \frac{C_d(\text{obs}) - C_d(\text{pre})}{C_d(\text{obs})} \times 100$$  \hspace{1cm} (18)

Where $C_d(\text{obs})$ = observed discharge coefficient and $C_d(\text{pre})$ = predicted discharge coefficient.

CONCLUSION:
Based on the limitations imposed on this experimental study, it can be concluded that:

The weir angle $\theta_2$ has an effect on the coefficient of discharge $C_d$ i.e. the coefficient of discharge $C_d$ increases with increase in the value of $\frac{H}{D}$. Also, the trend of variation of $C_d$ with $Q_{\text{theo}}$ is the same as that with $\frac{H}{D}$ and $\frac{h}{D}$. The average coefficient of discharge $C_{\text{ave}}$ increases with increase in the weir angle $\frac{\theta_2}{2}$ that is the wider the angle the higher the average coefficient of discharge and the narrower the angle the lower the average coefficient of discharge. As such the weir model with $\frac{\theta_2}{2} = 60^\circ$
has the highest value of $C_{dave} = 0.49934$. Hence the most efficient.

REFERENCES


Weir over Flow and Gate under Flow Rate” ”, International Journal of Science and Engineering Research, Vol.4, Issue4, April-2013